A NON-PARAMETRIC COINTEGRATION TEST OF PURCHASING POWER PARITY
THE CASE OF MALAYSIA

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ABSTRACT

This study employs the Johansen and Juselius (1990) cointegration test and the recently proposed Bierens (1997) nonparametric cointegration methodology to test the purchasing power parity (PPP) hypothesis for the Malaysian economies, with respect to her major trading partners- the United States, Japan and Singapore. The Bierens's non-parametric cointegration method is utilized in views of the superiority of non-parametric method at detecting cointegration when the data generating process is non-linear. Using the Johansen and Juselius cointegration approach, the evidence does not support the PPP proposition for all cases under investigate. Further analysis using the Bierens’s method, however, provides strong support for the PPP hypothesis for the Malaysian economies, with respect to U.S., Japan and Singapore. Since the Bierens’s method allows for non-linearity in the data generating process, the discrepancy between the findings from both techniques is interpreted as a consequence of significant non-linearity in the real exchange rate adjustment to PPP.

Keywords: Cointegration test; Nonparametric cointegration test; Purchasing Power Parity; Malaysian economies; Non-linearity.

INTRODUCTION

The oldest method of defining long-term exchange rate equilibrium is the purchasing power parity (PPP). The PPP theory simply states that the exchange rate between currencies of two countries should be equal to the ratio of the countries’ price level. Although in the short run, deviation of exchange rate from PPP might occur\(^1\), most economic theories suggest that PPP should hold in the long run.

Over the years, there has been an explosion of empirical research on the validity of PPP in the real world\(^2\). Two widely employed empirical tests for PPP are the unit root methodology and cointegration analysis. A necessary condition for PPP to hold in the long run is that the real exchange rate must be covariance stationary. The standard method for detecting non-stationary behaviour in a time series is to test for the presence of a unit root. Thus, rejection of a unit root in real exchange rate provides evidence supporting PPP. Another stream of literature is based on the cointegration technique. To provide empirical support for PPP, both the bilateral exchange rate and relative prices must be cointegrated and fail to reject the [1-1] cointegrating null.

Generally, empirical studies on PPP have yielded contradictory results. Even in the case of Malaysia, the results have been mixed, creating a debate among policy makers on the usefulness of the empirical findings. Study by Baharumshah and Ariff (1997) using unit root and Engle and Granger (1987)

\(^1\)The deviation can due to factors such as transaction costs, price rigidity, the differential composition of market baskets and prices indices, and imperfect markets (as results of subsidy, taxation, trade barriers, foreign exchange market interventions and the like).

\(^2\)Taylor (1995), Rogoff (1996) and Edison et al (1997) have done an excellent surveys on the empirical literature of PPP.
cointegration approach rejected the PPP proposition for Malaysian ringgit in terms of the U.S. dollar. Further analysis using the Johansen and Juselius (1990) multivariate approach also failed to support the PPP hypothesis. Bahmani-Oskooee (1993) who used the Engle and Granger procedure has found evidence in favour of weak form PPP between Malaysian and U.S. economies. On the other hand, a recent study by Bahmani-Oskooee and Mirza (2000) failed to support the mean reversion in real effective exchange rates for Malaysia by using the conventional ADF and KPSS unit root tests. To take into account the presence of structural breaks, Aggarwal et al (2000) employed both the single and multiple breaks unit root tests to test the validity of PPP for Malaysian currency in terms of Japanese yen. They found clear and significant evidence in favour of long run PPP. Another recent study by Azali et al. (2001) showed that PPP does hold in the long run between Japan and ASEAN economies using panel unit root and panel cointegration, which is in contrast with their results in the same study utilizing the Johansen and Juselius cointegration approach.

One possible explanation for the contrasting empirical evidence on the PPP hypothesis is that earlier studies generally make an implicit assumption that exchange rate behaviour is linear in nature (Taylor and Peel, 1997). With abounding empirical evidence supporting the presence of non-linearity in exchange rate time series data (see, for example, Hsieh, 1989; De Grauwe et al., 1993; Steurer, 1995; Brooks, 1996; Mahajan and Wagner, 1999), many researchers started asking themselves to what extent one should trust the results of linear methods like the conventional unit root tests and cointegration tests if the underlying data generating process is non-linear. Taylor and Peel (1997) and Sarno (2000), amongst others, illustrated that the adoption of linear stationarity tests is inappropriate in detecting mean reversion if the true data generating process of exchange rate is in fact a stationary nonlinear process. On the other hand, the Monte Carlo simulation evidence in Bierens (1997) indicated that the standard linear cointegration framework presents a mis-specification problem when the true nature of the adjustment process is non-linear and the speed of adjustment varies with the magnitude of the disequilibrium.

Due to the growing views that the world is non-linearly dynamics (Pesaran and Potter, 1993; Campbell et al., 1997; Barnett and Serletis, 2000), recent work in the PPP literature have attempted to address the issue of non-linearity in mean reversion. Serletis and Gogas (2000) applied non-linear techniques to test for non-linearity in real exchange rate series and found evidence that the behaviour of real exchange rate series under investigate are governed by non-linear dynamics. Other studies like Micheal et al. (1997), Sarno (2000) and Baum et al. (2001) employed nonlinear models such as the threshold autoregressive (TAR), smooth transition autoregressive (STAR) and exponential smooth transition autoregressive (ESTAR) models to model the behaviour of real exchange rates. All these studies provided strong support for the validity of long run PPP, in which the real exchange rate adjusts non-linearly towards its equilibrium PPP level.

The main objective of this study is to test the PPP hypothesis for the Malaysian economies with respect to her major trading partners. We utilize the Johansen and Juselius (1990) cointegration test and Bierens’s (1997) non-parametric cointegration test since both tests are in the same spirit, as pointed out by Bierens (1997). Therefore, in principle, both approaches should generate a similar outcome. However, if the data generating process is non-linear, the Johansen and Juselius approach may fail to detect any cointegrating relationship. This situation does not pose any problem for the Bierens’s method since it allows for non-linearity in the data generating process. In other words, the Bierens’ method is able to detect cointegration relationship regardless of the underlying data generating process.

This paper is organized as follows. Following this introduction, a brief description on the methodology used in this study is given. This is followed in Section III by a discussion on the empirical results. Concluding remarks are given at the end of the paper.
METHODOLOGY

The PPP hypothesis states that the nominal exchange rate (in domestic currency per foreign) should be equal to the ratio of domestic to foreign prices as

\[ E = \frac{P}{P^*} \tag{1} \]

where \( \lambda \) is a constant, \( E \) is the domestic currency per unit of foreign currency, \( P \) and \( P^* \) are the domestic and foreign price indices respectively. If PPP holds, the deviation from long-run PPP: \( R = \frac{P^*}{E} \), where \( R \) is the real exchange rate, should imply a stationarity process or that shocks have no permanent effect. In the methodology of cointegration, long-run PPP is implied by a cointegration relationship between nominal exchange rate and relative prices, with the cointegrating vector being \([1, -1]\). This study uses both the Johansen and Juselius (1990) and Bierens’s (1997) non-parametric cointegration tests to examine the long-run PPP hypothesis.

Johansen and Juselius (1990) Procedure

The Johansen and Juselius (JJ) (1990) multivariate cointegration technique uses maximum likelihood procedures to determine the number of cointegrating vectors among a vector of time series. Assume that \( y_t \) is modelled as a vector autoregression (VAR):

\[ y_t = \beta y_{t-1} + \ldots + \beta y_{t-k} + \mu_t \tag{2} \]

where \( y_t \) is a column vector of 2 endogenous variables. Equation (2) can be transformed into first-difference form as follows:

\[ \Delta y_t = \sum_{j=1}^{k-1} \Gamma_j \Delta y_{t-j} + \Delta y_{t-k} + \mu_t \tag{3} \]

where \( \Delta \) is the long-run relationships between the variables in \( y_t \) process. The estimation of the cointegrating vectors also determine from the matrix of \( \beta \), which can be written as:

\[ \beta = \Lambda \beta' \tag{4} \]

where \( \beta' \) is the \((r \times p)\) matrix of cointegrating vectors and \( \Lambda \) is the \((p \times r)\) matrix of error correction parameters that measure the speed of adjustment in \( \Delta y_t \). Since the rank of \( \beta \) is related to the number of cointegrating vectors, thus, if the rank of \( \beta \) equals to \( p \) or full rank, then \( y_t \) is a stationary process. If the rank of \( \beta \) is \( 0 < r < p \), implying that there are \( r \) cointegrating vectors and hence the group of time series contain a \((p-r)\) common trends. However, if the rank of \( \beta \) is zero, then the variables in \( y_t \) are non-cointegrated. Here, two likelihood ratio (LR) test statistics, namely the trace and maximum eigenvalue statistics are used to investigate the number of the cointegrating vectors. The trace statistics tests the \( H_0(r) \) against \( H_1(p) \) is written as:

\[ \text{Trace} = -T \sum_{r=1}^{p} \ln(1- \hat{\lambda}_r) \tag{5} \]

and the maximum eigenvalue statistic tests the \( H_0(r) \) against \( H_1(r+1) \) is given by:

\[ \text{Maximum eigenvalue} = -T \ln(1- \hat{\lambda}_{r+1}) \tag{6} \]

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In testing for long-run PPP under its bivariate case, nominal exchange rate and relative prices must exhibit only one cointegrating vector (p-1) or one common trend, thus imposing a test of restrictions on coefficients of JJ long-run cointegrating vector in the form of [1, -1]. If the restriction test fails to reject the null hypothesis, then nominal exchange rate will move one-by-one with relative prices, suggesting that PPP holds in the long-run.

**Bierens (1997) Non-parametric Cointegration Test**

The Bierens non-parametric cointegration test considers the general framework as:

\[ z_t = \pi_0 + \pi_1 t + y_t \]  

where \( \pi_0(q \times 1) \) and \( \pi_1(q \times 1) \) are optimal mean and trend terms, and \( y_t \) is a zero-mean unobservable process such that \( \Delta y_t \) is stationary and ergodic. The general framework assumes that \( z_t \) is observable q-variate process for \( t = 0, 1, 2, \ldots \ldots, n \).

Apart from some mild regularity conditions, or estimation of structural and/or nuisance parameters, further specification of the data-generating process for \( z_t \) are not required and thus this test is completely non-parametric.

The Bierens’s method is based on the generalized eigenvalues of matrices \( A_m \) and \( \begin{pmatrix} B_m + n & -1 \\ -1 & A_m \end{pmatrix} \), where \( A_m \) and \( B_m \) are defined in the following matrices:

\[
A_m = \frac{8\pi^2}{n} \sum_{k=1}^{m} k^{-2} \left( \frac{1}{n} \sum_{t=1}^{n} \cos(2k\pi(t - 0.5)/n)z_t \right)^2 \times \left( \frac{1}{n} \sum_{t=1}^{n} \cos(2k\pi(t - 0.5)/n)z_t \right) \\
B_m = 2n \sum_{k=1}^{m} \left( \frac{1}{n} \sum_{t=1}^{n} \cos(2k\pi(t - 0.5)/n)\Delta z_t \right)^2 \times \left( \frac{1}{n} \sum_{t=1}^{n} \cos(2k\pi(t - 0.5)/n)\Delta z_t \right)
\]

which are computed as sums of outerproducts of weighted means of \( z_t \) and \( \Delta z_t \), and \( n \) is the sample size. To ensure invariance of the test statistics to drift terms, the weight functions of \( \cos(2k\pi(t - 0.5)/n) \) is recommended here.

Similar to the properties of the Johansen and Juselius LR method, the ordered generalized eigenvalues of this non-parametric method are obtained as solution of the problem \( \text{det}[P_n - \lambda Q_n] = 0 \) when the pair of random matrices \( P_n = A_m \) and \( Q_n = \begin{pmatrix} B_m + n & -1 \\ -1 & A_m \end{pmatrix} \) are defined. Thus it can be used for testing hypotheses about the cointegration rank \( r \).

To estimate \( r \), two test statistics are imposed. First, Bierens (1997) derives the ‘lambda-min’ (\( \lambda_{\text{min}} \)), \( \hat{\lambda}_{n-r_0,m} \) which corresponds to the Johansen’s Max-L test to test for the hypothesis of \( H_0(r) \) against the \( H_1(r + 1) \) and tabulates the critical values for this test.
Second, Bierens’s approach also provides the \( g_m(r) \) which is computed from the Bierens’s generalized eigenvalues:

\[
\hat{g}_m(r) = \left( \prod_{k=1}^{q} \hat{\lambda}_{k,m} \right)^{-1} \quad \text{if } r = 0
\]

\[
= \left( \prod_{k=1}^{q-r} \hat{\lambda}_{k,m} \right)^{-1} \left( n^{-r} \prod_{k=n-r}^{q} \hat{\lambda}_{k,m} \right) \quad \text{if } r = 1, \ldots, n - 1
\]

\[
= n^{2r} \prod_{k=1}^{q} \hat{\lambda}_{k,m} \quad \text{if } r = n
\]

This statistic uses the tabulated optimal values (see Bierens, 1997, Table 1) for \( r \), provided \( r > q \), and \( m = q \) is chosen when \( r = n \). Then \( \hat{g}_m(r) \) converges in probability to infinity if the true number of cointegrating vector is unequal to \( r \), and \( \hat{g}_m(r) = \text{Op}(1) \) if the true number of cointegrating vector is equal to \( r \). Therefore, we have \( \lim_{n \to \infty} P(\hat{r}_m = r) = 1 \), when \( \hat{r}_m = \arg \min_{0 \leq r \leq 1} \{ \hat{g}_m(r) \} \). Thus, this test statistic is useful as a tool to double-check on the determination of \( r \).

Finally, a linear restriction on the cointegrating vectors in the form of \([1, -1]\) is needed to test for long-run PPP. For this purpose, the trace and lambda-max statistics are proposed by Bierens. The critical values of trace \((m = 2q, F(x) = \cos(2k\pi x))\) and lambda-max tests \((m = 2q, F(x) = \cos(2k\pi x))\) are given in Bierens (1997, Tables 3 and 4).

EMPIRICAL RESULTS

The Data

This study is based on monthly data from 1974:1 to 2002:5 for Malaysia, with her major trading partners’ data as reference. Specifically, the major trading partners of Malaysia are the United States, Japan and Singapore (see Economic Report 2003, Table 2.15, page 58). The consumer price indices are used to construct the relative prices, that is the ratio of domestic (Malaysia) to foreign (U.S., Japan and Singapore) prices. The nominal exchange rates are expressed as units of Malaysian ringgit per foreign currency (U.S. dollar, Japanese yen and Singaporean dollar). All the data used in this study are obtained from the International Financial Statistics database published by the International Monetary Fund. Both the nominal exchange rates and relative prices are transformed into logarithm form.

Stationarity Tests

To test for non-stationary behaviour in the time series of nominal exchange rates and relative prices, we use the non-parametric Philips-Perron (PP) (1988) \( \rho \)-test and non-parametric Bierens and Guo (1993) Cauchy test \#3 (BG3). The null hypotheses for both the PP and BG3 tests are nonstationarity and stationarity respectively. Table 1 and 2 report the results of the stationarity tests on both the nominal exchange rates and relative prices. The results clearly indicate that both variables are not stationary in the level but are able to attain stationary in the first-differences. In other words, all nominal exchange rates and relative prices are integrated of order one \( I(1) \). With these findings, we can proceed with the cointegration tests to check the validity of the long-run PPP hypothesis.
TABLE 1
Unit Root Tests for Nominal Exchange Rates

<table>
<thead>
<tr>
<th></th>
<th>Level Data</th>
<th>First-difference Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PP</td>
<td>BG-3</td>
</tr>
<tr>
<td><strong>Malaysian ringgit in terms of:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. dollar</td>
<td>-1.6383</td>
<td>15.1406*</td>
</tr>
<tr>
<td></td>
<td>(0.7410)</td>
<td>(0.04199)</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>-1.1660</td>
<td>59.8976*</td>
</tr>
<tr>
<td></td>
<td>(0.8140)</td>
<td>(0.01063)</td>
</tr>
<tr>
<td>Singaporean dollar</td>
<td>-0.0789</td>
<td>98.5459**</td>
</tr>
<tr>
<td></td>
<td>(0.8600)</td>
<td>(0.00646)</td>
</tr>
</tbody>
</table>

Note: * and ** denote significant at the 5% and 1% levels respectively.

TABLE 2
Unit Root Tests for Relative Prices

<table>
<thead>
<tr>
<th></th>
<th>Level Data</th>
<th>First-difference Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PP</td>
<td>BG-3</td>
</tr>
<tr>
<td>USA</td>
<td>-3.5615</td>
<td>115.5459**</td>
</tr>
<tr>
<td></td>
<td>(0.6790)</td>
<td>(0.00551)</td>
</tr>
<tr>
<td>Japan</td>
<td>1.4366</td>
<td>80.7636**</td>
</tr>
<tr>
<td></td>
<td>(1.0000)</td>
<td>(0.00788)</td>
</tr>
<tr>
<td>Singapore</td>
<td>-0.0379</td>
<td>139.6698**</td>
</tr>
<tr>
<td></td>
<td>(0.9890)</td>
<td>(0.00456)</td>
</tr>
</tbody>
</table>

Note: ** denotes significant at the 1% level.

Johansen and Juselius (1990) Cointegration Test
Under the Johansen and Juselius method, we use the Akaike Information Criterion (AIC) to determine the order of the VAR model. Table 3 reports the trace and maximal statistics. Both statistics are then used to determine the number of cointegrating vectors. In this bivariate case, if both series are cointegrated, we proceed by imposing the restriction \([1, -1]\) on the cointegrating vector. The acceptance of the null hypothesis means that the long-run PPP holds.

Results summarized in Table 3 show that both the trace and maximal eigenvalue statistics reject the null hypothesis of no cointegrating vector \((r = 0)\) for all cases under investigate. Thus, there is a need to impose the restriction \([1, -1]\) on the cointegrating vector. With this \([1, -1]\) restriction, we reject the null hypothesis in all cases, which provide evidence against the PPP proposition.

TABLE 3
Cointegration Test Based on Johansen and Juselius Approach

<table>
<thead>
<tr>
<th></th>
<th>Lag</th>
<th>(\lambda)-max</th>
<th>(\lambda)-trace</th>
<th>(\beta^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(H_0: r = 0)</td>
<td>(H_0: \lambda \leq 1)</td>
<td>(H_0: r = 0)</td>
</tr>
<tr>
<td>USA</td>
<td>2</td>
<td>15.7*</td>
<td>1.3</td>
<td>17.0*</td>
</tr>
<tr>
<td>Japan</td>
<td>1</td>
<td>21.7*</td>
<td>0.4</td>
<td>22.0*</td>
</tr>
<tr>
<td>Singapore</td>
<td>2</td>
<td>14.9*</td>
<td>0.0</td>
<td>14.9</td>
</tr>
</tbody>
</table>

Notes: The critical values for the trace and maximal eigenvalue statistics are tabulated in Osterwald-Lenum (1992). * denotes significant at the 5% level.
Bierens (1997) Non-parametric Cointegration Test

Further analysis using the Bierens’s non-parametric cointegration method can serve to check the robustness of the Johansen’s method in views of the superiority of non-parametric method at detecting cointegration when the data generating process is non-linear. Table 4 reports the results of the Bierens’s test. The results provide evidence of cointegration for the Malaysian real exchange rates in all cases, a result which is consistent with the Johansen and Juselius approach. However, by imposing the [1, -1] restriction, the null hypothesis cannot be rejected. These evidence of mean reversion in the Malaysian real exchange rates with respect to all her trading partners are in sharp contrast with our earlier findings obtained from Johansen and Juselius method.

TABLE 4
Cointegration Test Based on Bierens Approach

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{min}$</th>
<th>$g_n(r_0)$</th>
<th>$\beta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_0$: $r = 0$</td>
<td>H$_0$: $r = 1$</td>
<td>$r_0 = 0, 1, 2$</td>
<td>$H_0: \beta' = [1, -1]$</td>
</tr>
<tr>
<td>USA</td>
<td>0.00395*</td>
<td>0.84960</td>
<td>31.4582926E+001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>50.90829893E+001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>42.47888530E+006</td>
</tr>
<tr>
<td>Japan</td>
<td>0.04083</td>
<td>0.09764</td>
<td>47.54002518E+002</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>25.50346714E+002</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>28.10970324E+005</td>
</tr>
<tr>
<td>Singapore</td>
<td>0.00005*</td>
<td>0.01542*</td>
<td>40.33059065E+005</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12.04985490E+001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>33.13455069E+002</td>
</tr>
</tbody>
</table>

Note: * denotes significant at the 5% level.

As pointed out by Bierens (1997), the Bierens’s method is in the same spirit with Johansen and Juselius (1990) approach. Therefore, in principle, both approaches should generate a similar outcome. Since the Bierens’s method allows for non-linearity in the data generating process, the discrepancy between the findings from both approaches is interpreted as a consequence of significant non-linearity in the real exchange rate adjustment to PPP. Coakley and Fuertes (2001) explained that the nonlinear adjustment process in the real exchange rate would cause the standard linear cointegration approach to present a mis-specification problem. In other words, the failure of Johansen and Juselius method to establish real exchange rate stationarity does not necessarily invalidate the long run PPP. Instead, it is the presence of non-linearity that contributes to its poor performance at detecting cointegration. Our empirical findings also corroborate those of Liew et al. (2002) which provide empirical support that the Malaysian real exchange rate adjusts non-linearly towards its PPP equilibrium level.

CONCLUSIONS

This study employs the Johansen and Juselius (1990) cointegration test and the recently proposed Bierens (1997) nonparametric cointegration methodology to test the purchasing power parity (PPP) hypothesis for the Malaysian economies, with respect to her major trading partners- the United States, Japan and Singapore. The Bierens's non-parametric cointegration method is utilized in views of the superiority of non-parametric method at detecting cointegration when the data generating process is non-linear. This is further motivated by the empirical evidence that non-linearity abounds in exchange rate time series data, coupled with the growing literature suggesting real exchange rate adjusts non-linearly towards its equilibrium PPP level.
Using the Johansen and Juselius cointegration approach, the evidence does not support the PPP proposition for all cases under investigation. Further analysis using the Bierens’s method, however, provides strong support for the PPP hypothesis for the Malaysian economies, with respect to U.S., Japan and Singapore. Since the Bierens’s method allows for non-linearity in the data generating process, the discrepancy between the findings from both techniques is interpreted as a consequence of significant non-linearity in the real exchange rate adjustment to PPP. As pointed out by Coakley and Fuertes (2001), the nonlinear adjustment process in the real exchange rate would cause the standard linear cointegration approach of Johansen and Juselius to present a mis-specification problem. In other words, the failure of Johansen and Juselius method to establish real exchange rate stationarity does not necessarily invalidate the long run PPP. Instead, it is the presence of non-linearity that contributes to its poor performance at detecting cointegration. Our empirical findings also corroborate those of Liew et al. (2002) which provide empirical support that the Malaysian real exchange rate adjusts non-linearly towards its PPP equilibrium level.

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