NONLINEAR MEAN REVERSION IN REAL EXCHANGE RATES: EVIDENCE FROM THE ASEAN-4

Ahmad Zubaidi Baharumshah
Liew Khim Sen
Evan Lau

ABSTRACT

Utilizing formal nonlinear unit root test (Sarno, The behavior of US public debt: a nonlinear perspective. Economics Letters 2001: 119 – 125), this study provides robust evidence of nonlinear mean reversion in the real exchange rates of 4 major ASEAN countries. We conclude that the bulk of the evidence based on conventional unit root tests may be biased against long run Purchasing Power Parity (PPP)

Keywords: Purchasing power parity; Real exchange rate; ASEAN; Nonlinear unit root test; STAR model.

JEL classification: F31, F32

INTRODUCTION

A number of recent empirical studies have provided evidence that the behavior of exchange rate is in fact nonlinear in nature (Micheal et al., 1997; Taylor and Peel; 1997; Sarno, 1998; Sarno, 2000a,b; Sarantis, 1999; Taylor and Peel; 2000; Baum et al., 2001; Liew et al., 2002). Through performing formal linearity test procedure (Luukkonen et al., 1988; Granger and Terasvirta, 1993) these studies have found enough evidence to reject the null of linearity in empirical exchange rate series. In particular, most of these studies conclude that exchange rate adjustment varies nonlinearly with respect to the size of deviation, and that this adjustment is well characterized by the smooth transition autoregressive (STAR) process. STAR model is an advance econometric model that is deemed most appropriate in capturing the behavior of exchange rate, which adjust every moment but the speed of adjustment depends on the variations of exchange rate from its equilibrium level. Under the assumption of this STAR model, exchange rate is globally stable (mean reversion) due to its strong tendency to return to its equilibrium level, although nonstationarity (non-mean reversion) may be detected locally. Global stability of exchange rate has been verified empirically using sample data of various rates. Sarno (2000b) for instance, found empirical evidence that deviations of Purchasing Power Parity (PPP) revert to a constant equilibrium level in a nonlinear fashion, in the context of the real exchange rate behavior in the Middle Eastern country. In a similar study on the exchange rates of developed countries, Baum et al. (2001) arrive at similar conclusion. Liew et al. (2002) also report the nonlinear adjustment of Asian nominal exchange rate deviation towards PPP. The consistent finding of these extensive studies suggests that the existence of nonlinearities in exchange rates is now undisputable. Nevertheless, none of these studies conduct a straightforward and more robust test in detecting the stationarity of exchange rate. Inspire by this motivation, this study investigates the mean-reversion behavior of the selected Asian real exchange rate using the nonlinear unit root test formulated by Sarno (2001). Lately, Sarno extends the standard Augmented Dickey and Fuller (ADF) test to its nonlinear version, based empirical fact that exchange rate adjustment follows the STAR process. This new nonlinear test has several attractive features. First, following from the STAR

1 For small deviations from equilibrium, exchange rate may adjust very slowly or not adjust at all but for large deviations, it adjusts rapidly to its equilibrium. These findings are consistent with the theoretical no-arbitrage model proposed by Dumas (1992), which suggests that there is a band of inaction due to transaction cost.

2 See Dijk et al, 2001 for a comprehensive review on the specification and estimation this model and its variants.

3 Sarno (2001) however, applies this test in the context of US debt-GDP series.
specification, Sarno’s nonlinear unit root test (for brevity, we refer this as Sarno test hereafter) does not impose local stationarity in the series under study. Thus, it has greater power in detecting the mean reversion behavior over the conventional ADF, especially when the true data generating process of the series is actually nonlinear in nature. In other words, ADF test may fail to detect a globally mean reversion series and mistaken it as nonstationary. Second, Sarno test is easily conducted and unlike STAR estimation, whereby the significance of nonlinear parameters such as transition parameter has been indirectly regarded as supportive of nonlinear adjustment behavior, interpretation of Sarno test is straightforward and thus more robust. Third, the null hypothesis of linearity of nonlinearity in the series of interest using standard linearity tests does not necessarily imply mean-reversion. Thus, further examination of the mean reversion behavior need to be done separately. Sarno test seems to be an appropriate complementary to the linearity test. More interestingly, Sarno test may also be conducted independently as rejection of the null of nonlinear nonstationary implies not only the existence of nonlinear behavior, but also mean reversion. Fourth, as the ADF test is nested in Sarno test, application of the former test assuming in priori that the series under study is linear in nature may lead to misspecification. This implies that ADF test can only be conducted upon certain valid restrictions. Despite the attractive advantages of Sarno test over the relevant existing tests, to the best of our effort, we fail to search for any published article applying this test in the context of exchange rate. Thus, the contributions of this study, among others, is to provide empirical evidence of nonlinear mean reversion of exchange rate, if any, based on formal nonlinear unit root test of Sarno (2001).

The rest of this paper is outlined as follows. Section 2 describes the nonlinear unit root test, whereas the empirical analysis and bootstrap evidence of this study is presented in Section 3. The final section concludes this study.

**ESTIMATION PROCEDURES OF NONLINEAR UNIT ROOT TEST**

Sarno (2001) prudently reparameterizes the commonly used STAR model in the first difference version into the nonlinear unit root test analogue to conventional augmented Dickey-Fuller (ADF) unit root test. The resulting test can be expressed as

\[ \Delta y_t = \alpha + \rho y_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta y_{t-i} + (\alpha^* + \rho^* y_{t-1} + \sum_{i=1}^{p-1} \beta_i^* \Delta y_{t-i}) F(\bullet) + \epsilon_t \]  

(1)

where \( \Delta z_j = z_j - z_{j-1} \); \( p > 0 \) is the autoregressive order; \( F(\bullet) \) stand for transition function; and \( \epsilon_t \sim i.i.d(0, \sigma^2) \). Sarno (2001) adopts the commonly used exponential specification of \( F(\bullet) \) in the nonlinearity study, given by

\[ F(\bullet) = 1 - \exp[-\gamma^2 (y_{t-d} - u)^2 / \sigma_y^2] \]  

(2)

where \( \gamma^2 \), \( u \) and \( \sigma_y^2 \) are transition parameter, equilibrium level and sample variance of \( y \) series respectively, and \( d > 0 \) is the delay lag length.

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4 For instance, the commonly applied linearity test of Lukkonen et al. (1988), and the BDS test developed by Brock et al. (1996).

5 Mathematically expressed as \( \Delta y_t = \tilde{\alpha} + \tilde{\rho} y_{t-1} + \sum_{i=1}^{p-1} \tilde{\beta}_i \Delta y_{t-i} + \epsilon_t \)  

(3) where \( \Delta z_j = z_j - z_{j-1} \); \( p > 0 \) is the autoregressive order; \( \epsilon_t \sim \tilde{\alpha} \) (0, \( \sigma^2 \)). Rejection of the unit root null hypothesis \( H_0 : \tilde{\rho} = 0 \) against the stationary linear alternative hypothesis \( H_1 : \tilde{\rho} < 0 \) implies linear mean reversion.

6 The exponential transition function as specified in (3) has a symmetrically bell-shaped distribution around the equilibrium level, \( u \) and is bounded between 0 and 1. See Granger and Teräsvirta (1993) and Dijk et al. (2001), among others, for more details on the properties of this function.
In the nonlinear model (1) the parameters of interest for the stability of $y_t$ are $\rho$ and $\rho^*$. Assuming that the true process for $y_t$ is given by the nonlinear model (1), estimates of the parameter $\rho$ may exceed 0 in the case of locally nonstationary series, but one must have $(\rho + \rho^*) < 0$ in order to meet the globally stable requirement\(^7\). As such, estimated $\hat{\rho}$ in (1) will tend to lie between $\rho$ and $(\rho + \rho^*)$, depending upon the distribution of observed deviations from the equilibrium level $u$. Hence, Sarno (2001) exerts that the unit root null hypothesis $H_0 : \rho = 0$ may not be rejected against the stationary linear alternative hypothesis $H_1 : \rho < 0$, although the true nonlinear process is globally stable with $(\rho + \rho^*) < 0$. Estimation of (1) with $F(\bullet)$ as specified in (2) may be carried out using nonlinear least squares (NLS).

EMPIRICAL ANALYSIS AND BOOTSTRAP EVIDENCE

Five major US dollar based ASEAN real exchange rates in their logarithmic form are considered in this study. These real exchange rates are derived from the relative form of PPP hypothesis, namely $y_t = s_t + p_t^* - p_t$, where $y_t$ is the logarithm of nominal exchange rate (domestic price of foreign currency) at time $t$, and $p_t^*$ and $p_t$ are the logarithms of foreign and domestic price levels respectively. This specification of real exchange rate is effectively the deviation of nominal exchange rate from the PPP equilibrium (Sarno, 2000b) and thus the mean reversion of real exchange rate may be regarded as the validity of long-run PPP\(^8\). The data set comprises of end-of-period seasonally unadjusted nominal bilateral exchange rates for the domestic currency vis-a-vis the US dollar for Indonesia (INR/USD), Malaysia (MYR/USD), Philippines (PHP/USD), Singapore (SGD/USD) and Thailand (THB/USD), as well as the relevant Consumer Price Indices (CPI). All data are obtained from International Financial Statistics recorded by International Monetary Fund at quarterly interval from 1968:1 to 2001:2.

Standard ADF unit root test (3) has been conducted for the logarithm of the nominal exchange rates. Results suggest that all nominal exchange rates are non-mean reversion and are integrated of order one. The CPI series are also integrated of order one by the same test. Further analysis shows that exchange rates are not cointegrated with their respective relative prices on the basis of Johansen and Juselius (1989) cointegration test procedure. To conserve space, these results are not reported but are available upon request.

The augmented first order of Lukkonen et al. (1988) linearity test procedure is performed to test the linearity nature of the real exchange rates. This test requires that the following auxiliary regression to be carried out:

$$y_t = c + \sum_{i=1}^{p} (g_i y_{t-i} + g_i^* y_{t-i}^* + h_i^* y_{t-i-d}^*) + \mu_t$$

where $c$, $g_i$, $g_i^*$, and $h_i^*$ for $i = 1, ..., p$ are parameters to be estimated and $\mu_t \sim iid(0, \sigma^2)$.\(^4\)

The null hypothesis of $H_0 : g_i^* = h_i^* = 0$ for $i = 1, ..., p$ implies that the series is linear in nature. If the overall significance of the nonlinear coefficients $g_i^*$'s and $h_i^*$'s, has been rejected based on certain test statistics such as the $F$ statistic, one may conclude that the series is nonlinear in nature in favor of the STAR process (Teräsvirta, 1994).

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\(^7\)This means failing to reject the null of $(\rho + \rho^*) = 0$ in (1) conditioned on $\rho > 0$ implies strong evidence non-mean reversion (in both linear and nonlinear perspective).

\(^8\)In our opinion, the beauty of conducting nonlinear unit root test on real exchange rate is that once nonlinear mean reversion is found, we could safely conclude that the nominal exchange rates and the relative prices concerned are nonlinearly cointegrated. Formal nonlinear cointegration test, which is still unformulated, can thus be bypassed.
In this study, the optimal autoregressive order $p$ is determined by examining the partial autocorrelation function (PACF) of the series, whereas the optimal delay parameter $d$ is selected by performing (4) for $1 \leq d \leq 12$ and choose the one that minimizes the marginal significance (or $p$) value of the $F$ statistic of the linearity test (Liew et al., 2002). The PACF of the real exchange rates are plotted in Figure 1. Figure 1 clearly suggests that $p$ equals 1 are adequate for all these rates.

The linearity test results are summarized in Table 1. Table 1 reveals that the null of linearity has been rejected at standard significance levels in favor of STAR model in all cases, implying that all real exchange rates under study adjust nonlinearly.

**FIGURE 1**

The Partial Autocorrelation Functions (PACF) of real exchange rates

![PACF plots for INR/USD, MYR/USD, PHP/USD, SGD/USD, THB/USD](image)

**TABLE 1**

<table>
<thead>
<tr>
<th>Real exchange rate</th>
<th>$p^a$</th>
<th>$d^b$</th>
<th>$F$ statistic</th>
<th>$p$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INR/USD</td>
<td>1</td>
<td>2</td>
<td>19.369</td>
<td>0.000</td>
</tr>
<tr>
<td>MYR/USD</td>
<td>1</td>
<td>1</td>
<td>5.427</td>
<td>0.021</td>
</tr>
<tr>
<td>PHP/USD</td>
<td>1</td>
<td>11</td>
<td>39.802</td>
<td>0.000</td>
</tr>
<tr>
<td>SGD/USD</td>
<td>1</td>
<td>8</td>
<td>12.693</td>
<td>0.001</td>
</tr>
<tr>
<td>THB/USD</td>
<td>1</td>
<td>11</td>
<td>8.926</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Notes:  

$^a$ The optimal autoregressive lag length $p$ is determined by inspecting the PACF of the series.  

$^b$ The optimal delay parameter $d$ is chosen from the one that minimizes the $p$ value of the implied $F$ statistic.
The results of Sarno’s procedure are tabulated in Table 2. Table 2 shows that \((\rho + \rho^*) < 0\) in all cases implying that real exchange rates are globally stable. We note here that the \(F\) specification test has been carried out and the null hypothesis linear unit root test procedure (3) has been rejected in favour of the nonlinear unit root test procedure (1) based (except MYR/USD case). All in all, this study has demonstrated that all the five major ASEAN real exchange rates are nonlinear in nature (by the Linearity test) and that this nonlinear adjustment is mean reverting (by the Sarno test, with the exception of MYR/USD).

**TABLE 2**

<table>
<thead>
<tr>
<th>Real exchange rate</th>
<th>(\rho)</th>
<th>(t)</th>
<th>(\rho^*)</th>
<th>(t)</th>
<th>(\rho + \rho^*)</th>
<th>(F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INR/USD</td>
<td>–3.261</td>
<td>–2.884</td>
<td>3.204</td>
<td>2.837</td>
<td>–0.056</td>
<td>29.023 [0.000]</td>
</tr>
<tr>
<td>MYR/USD</td>
<td>–0.203</td>
<td>–1.065</td>
<td>0.190</td>
<td>0.976</td>
<td>–0.012</td>
<td>0.654 [0.420]</td>
</tr>
<tr>
<td>PHP/USD</td>
<td>–0.744</td>
<td>–2.736</td>
<td>0.693</td>
<td>2.539</td>
<td>–0.051</td>
<td>8.120 [0.005]</td>
</tr>
<tr>
<td>SGD/USD</td>
<td>–0.259</td>
<td>–2.223</td>
<td>0.204</td>
<td>1.695</td>
<td>–0.055</td>
<td>6.942 [0.010]</td>
</tr>
<tr>
<td>THB/USD</td>
<td>0.479</td>
<td>2.053</td>
<td>–0.557</td>
<td>–2.379</td>
<td>–0.060</td>
<td>9.125 [0.003]</td>
</tr>
</tbody>
</table>

Notes: \(F\) tests for the null hypothesis that the linear specification (1) is correct. Rejection of the null hypothesis implies that the nonlinear specification (2) is correct.

**CONCLUSION**

In line with a number of recent studies (Micheal et. al., 1997; Taylor and Peel, 1997; Sarno; 1998; Sarno 2000a,b; Sarantis, 1999; Taylor and Peel, 2000; Baum et. al, 2001; Liew et. al. 2002), this study found that ASEAN real exchange rates adjust nonlinearly by performing the formal linearity test procedure. Nonetheless, the stationarity of this nonlinear adjustment has not been studied so far. To close this gap, this study utilises the formal nonlinear unit root test proposed by Sarno (2001) and found evidence of nonlinear mean reversion in four out of five selected ASEAN real exchange rates.

**REFERENCES**


