FORECASTING THE RINGGIT / YEN RATE USING EXPONENTIAL SMOOTH 
TRANSITION AUTOREGRESSION (ESTAR) MODEL

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Liew Khim Sen
Evan Lau

ABSTRACT

The purpose of our paper is to contribute to the debate on the relevance of nonlinear forecasts in the financial markets. To that end, this study forecasts the yen-based ringgit by using the using Exponential Smooth Transition Autoregressive (ESTAR) model. When formally tested for forecast accuracy, the results reveal that the ESTAR out-of-sample predictors statistically outperform both the linear AR and random walk models at standard significant levels. The hypothesis of equal forecasting accuracy between ESTAR models and the random walk model is formally rejected based on the Fisher sign test. This paper offers further evidence on the ability to forecast exchange rates using nonlinear methods. Hence, we conclude that linear models are not always the optimal for forecasting exchange rate. There is some forecast accuracy that can be gained by considering the information content in the residuals of the purchasing power parity (PPP).

Key words: ESTAR; exchange rate

JEL classification: C53; F31

INTRODUCTION

A large number of papers have compared the forecasting performance of exchange rate models, structural and non-structural, against the simple random walk model. Generally, the findings from these studies corroborate with the fact that it is extremely difficult to out-predict the random walk model. This pessimism about the forecasting ability of the exchange rate model is reflected in an influential paper published by Messe and Rogoff (1983). The authors performed a large number of statistical tests, indicating that it is difficult to beat the simple random walk model. However, attempts to refute the Messe and Rogoff (1983) results include Cheung (1993) (fractional integration model), Diebold and Nason (1990) (autoregressive nonparametric method), Meese and Rose (1991) (non-linear models), and more recently Chinn and Meese (1995) (non-linear functions and structural models) and Mark and Sul (1988) (panel regression approach). Indeed, these studies have found models that could outperform a random walk model in out-of-sample forecasting exercise.

In this paper, we argue that exchange rate is difficult to track because of the following reasons: First, poor forecasting performance could be due to nonlinearity in exchange rate series. Specifically, exchange rate adjusts in a nonlinear fashion to its long run equilibrium path. This view is in line with several authors that have shifted their focus to nonlinear models (Granger and Teräsvirta 1993, Teräsvirta 1994, Sarno (2000)). In fact, Sarno (2000) shows that there is strong evidence to suggest that deviation from exchange rate revert to the equilibrium in a nonlinear fashion for most of the currencies in the Middle Eastern countries. Second, significance in variability of exchange rates is possible because the rates are not driven by fundamental variables at all times. For example Groen (1998) finds that deterioration in the accuracy of out-of-sample monetary forecasts of the US dollar price of the yen, deutschmark and Swiss franc in the 1990s were due to two major events: banking

\(^1\) For surveys of this literature see Meese and Rogoff (1983), Diebold and Nason (1990), Meese and Rose (1991) and Lin and Chen (1998), among others.
crisis in Japan and the residual fiscal consequences from the German reunification. These events accordingly led to transient nonmonetary factors into the pricing of exchange rates.

In addressing this problem, we argue that the solution lies in the way one deals with the exchange rate volatility. This volatility that is entirely captured by the residuals or equivalently the forecast errors of the long-run equilibrium-based model may contain information sufficient to improve predictive power of the model (Shaffer 1988). Thus, rather than being whitened as in the conventional econometrics modelling process, we argue that the residuals should be retrieved and incorporated into the forecasting model to yield better forecast points.

The purpose of this paper is to provide further evidence on the ability to forecast exchange rates using nonlinear econometric methods. More precisely, this article attempts to address the question about whether exponential smooth transition autoregressive (ESTAR) predictors can improve out-of-sample forecasting in the Malaysian ringgit/yen (RMY/YPY) rate. To this end, we first estimate the residuals of the long-run equilibrium on the basis of purchasing power parity (PPP) hypothesis and then model the resulting PPP-based residuals with Exponential Smooth Transition Autoregressive (ESTAR) model. The ESTAR model is a variant of Smooth Transition Autoregressive (STAR) model, which is a nonlinear time series model that allows the variable under investigation to alternate between two distinct regimes with a smooth transition between these regimes. The model provides a promising alternative in modelling exchange rate series since there is now a growing consensus that the behaviour exchange rate is nonlinear in nature (Taylor and Peel 2000) and the references cited there in). Moreover, the ESTAR model is capable of generating a symmetry adjustment process and hence is suitable to estimate the response to positive and negative deviations from the exchange rate fundamental equilibrium, which is deemed symmetrical (Baum et al. 2001).

In spite of the usefulness of the ESTAR model in modelling the nonlinearities in the pricing of exchange rates, its application (STAR in general) is rather limited in the empirical literature. This may be partially due to its short history upon introduction and its complicated structure with unknown statistical properties. To the best of our knowledge, there are only few empirical studies on the STAR model, and we do not know of any study that uses similar approach to model the currencies of emerging markets, except for Sarno (2000). Most of articles are theoretical in nature and model estimations are generally for illustration purposes. In this study, we fit the ESTAR model to the MYR/JPY rate, a currency that took a sharp fall during the 1997/98 Asian financial crises. We use the ESTAR model to predict the MYR/JPY rate and evaluate the predictive power of the model, an issue ignored by the literature so far. This study completes the task by using the linear Autoregressive or AR (p) and random walk model as benchmarks. The results of the empirical investigation show

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2 A good review of the PPP model is found in, for instance, Balassa (1964), Clark et al. (1994), MacDonald (1997), Sarno (2000) and Baum et al. (2001). The last two articles deal with the nonlinear adjustment of exchange rate toward PPP.

3 Most earlier works, for instance, Chan and Tong (1986), Luukkonen et al. (1988), Saikkonen and Luukkonen (1988), Luukkonen (1990), Teräsvirta (1994) and Eirtheim and Teräsvirta (1996) discuss the theoretical issues on the linearity tests and model specification of STAR models. One exception is the article by Teräsvirta and Anderson (1993), that applies the STAR model to study the characteristics of business cycle. Few applications of STAR models in exchange rates including Taylor and Peel (2000), Sarno (2000) and Baum et al. (2001) have only emerge lately.

4 The encouraging empirical evidence on the nonlinear adjustment of exchange rate toward its fundamental equilibrium in the works of Taylor and Peel (1997, 2000), Sarno (2000) and Baum et al. (2001) suggests that nonlinear model might be a better estimation procedure for the ringgit.

5 To this end, Teräsvirta and Ande rson (1993) remain the only work that includes out-of-sample forecasting, but it is only in the field of business cycle.

6 These benchmarks are chosen for two reasons. First, the overall prediction performance of nonlinear, models should be an improvement upon the linear model (Tong and Lim, 1980). There is no point to model nonlinearities if the marginal gain from extending the linear AR (p) model is not significant. Hence the effort of iESTAR modelli ng is only paid off if it is capable of outperforming its linear competitor significantly. Second, in the past studies, empirical exchange rate models rarely beat the random walk (RW) model. Hence, we desperately desire to know whether ESTAR is capable of outperforming this simple ‘model of no change’.
that the estimated ESTAR models significantly improve over both the linear AR and random walk models up to fourteen quarters in predicting future values of MYR/JPY rate.

The contributions of this study are threefold. First, most of the research on exchange rate has concentrated on the industrialized countries with relatively little attention given to the currencies of the emerging economies. It is natural to extend the analysis to the emerging markets in order to enhance our understanding of the currency markets in these countries. Second, the present article fills the gap between model estimation and the out-of-sample forecasting performance of the ESTAR model. In this study we formally evaluate the out-of-sample performance of the ESTAR model against the standard linear benchmark models, which is rarely attempted in the earlier studies. Finally, the sampling period covers the 1997/98 Asian financial crises and the post-crisis era. This allows us to evaluate the robustness of our forecasts from the models during the turbulences of the currency market.

DATA AND METHODOLOGY

Preliminary data analysis

The bilateral exchange rate of the Malaysia ringgit / Japanese yen (MYR/JPY) and the relative price \( (P_t) \), which is constructed as the ratio consumer price index (CPI) of Malaysia to the CPI of Japan are utilized in this study. All data are collected from the International Monetary Fund’s International Financial Statistics (IMF/IFS). Our sample period covers from the first quarter of the year 1980 to the fourth quarter of the year 2000 (1980:1 to 2000:4). The whole sample period is divided into two portions. The first sub-period beginning in 1980:1 and ending in 1997:2 is used for estimating the model, while the remaining observations are kept for assessing the out-of-sample forecast performance.

All the series involved were tested for the order of integration using the commonly used Augmented Dickey-Fuller (ADF) and non-parametric Phillips-Perron (PP) unit root tests. The PP statistics accounts for non-independent and identically distributed processes by using a nonparametric adjustment to the standard Dickey-Fuller procedure. Since the test statistics are known to be sensitive to choice of a lag structure, the Akaike Information criteria is used to find the optimal lag length. The results of the unit root tests as according to Table 1 overwhelmingly suggest the null hypothesis of a unit root I(1) cannot be rejected for any of the test statistics over the sample period. On the other hand, the test statistics suggest that all variables are first difference stationary, which implies they are all I(1) variables. The conclusion from the unit root test suggests that exchange rate and relative price have roots of the same order and can be tested for cointegration.

Next, we proceed to investigate whether or not the long-run PPP conditions hold using the Johansen and Juselius (1990) cointegration multivariate approach. Results of the cointegration test based on trace test are depicted in Table 2. The lag length of the VAR model is chosen using the AIC criteria.
The test result provides strong evidence that MYR/JPY rate and relative price are cointegrated, as suggested by long-run PPP. To check whether exchange rate enters into the cointegrating relation, a log-likelihood ratio test of a zero restriction on the cointegrating vector is performed. The statistical evidence reveals that the cointegrating relationship includes the exchange rate and relative price is accepted at standard significance levels. Thus, our result seems to suggest that MYR/JPY rate adjusts for international differences in inflation rates in the long run. The estimated cointegrating vector for MYR/JPY rate is as follows:

$$\hat{X}_t = -3.6303 + 6.086P_t$$

where $$\hat{X}_t$$, $$P_t$$ and $$R^2$$ refer to the estimate exchange rate, relative price and coefficient of determination of the model. The t-statistic (in parentheses) with a single asterisk (*) implies the estimated coefficient concerned is significant at 1% level. It is worth pointing out here that the $$R^2$$ suggests that price ratio explains only about 52% of the fluctuation in the ringgit-yen rate, indicating that the information content of the PPP-based residuals need to be further analyzed. To conclude, the result is in line with other recent emerging consensus that PPP may hold after all (Taylor and Sarno 1998, Mohamed et al. 2001).

Table 2.

<table>
<thead>
<tr>
<th>Optimal Lag</th>
<th>Likelihood Ratio of Eigen Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r = 0</td>
</tr>
<tr>
<td>10</td>
<td>24.369*</td>
</tr>
<tr>
<td>Critical Values</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>19.90</td>
</tr>
<tr>
<td>1%</td>
<td>24.60</td>
</tr>
</tbody>
</table>

Notes: r denotes the hypothesized number of cointegrating equation. Optimum lag-length is determined by the Akaike Information Criterion (AIC). * denotes rejection of hypothesis at 5% significant level.

Specifications of ESTAR models

The PPP-based residuals are modelled using the following unrestricted ESTAR specification

$$z_t = \beta_0 + \sum_{i=1}^{p} \beta_i z_{t-i} + \left[ \sum_{i=1}^{p} \beta_i z_{t-i} \right] \left[ 1 - \exp \left( -\left( \gamma^2 / \hat{\sigma}^2 \right) z_{t-d} \right) \right] + \epsilon_t,$$

where $$z_t$$ denotes the zero-mean residuals of interest. $$\beta_i; i = 0, \ldots, p$$ and $$\beta_i; i = 1, \ldots, p$$ are the linear and nonlinear autoregressive parameters to be estimated. The transition parameter $$\gamma^2$$ is standardized by dividing with $$\hat{\sigma}^2$$, the variance of the residuals $$z_t$$. As usual, the residual is assumed as $$\epsilon_t \sim \text{i.i.d.} (0, \sigma^2)$$. Following Taylor and Peel (2000), the following restricted ESTAR specification is also estimated:

$$z_t = \psi_0 + \left[ \sum_{i=1}^{p} \psi_i z_{t-i} \right] \left[ \exp \left( -\left( \gamma^2 / \hat{\sigma}^2 \right) z_{t-d} \right) \right] + \epsilon_t$$

Other variables that may be included are interest rate differential, money supplies and stock prices as in Gan and Soon (2000) and Baharumshah et al. (2002) for instance. However, our main objective here is to keep the model as simple as possible.
where \( \psi_i; i = 0, 1, \ldots, p \) are parameters to be estimated.

The above specification is a special case of Equation 2 with the restrictions \( \sum_{i=0}^{p} \beta_i \leq 1 \) and \( \beta_i^* = -\beta_i \) for each \( i \). These restrictions imply an equilibrium level for \( z_i \) in the neighbourhood of which \( z_i \) is close to a second order unit root process, adjustment faster in the absolute size of the deviation from equilibrium. Both Equations 2 and 3 can be estimated using the nonlinear least square (NLS) method by employing the maximum likelihood method\(^8\).

**Linearity tests**

In this study, we employ the augmented first-order auxiliary linearity test (hereafter referred to as the LST test) due to Lukkonen, Saikkonen and Teräsvirta (1988) as well as the Lagrange Multiplier (LM) test proposed by Teräsvirta (1994) to determine the adequacy of the ESTAR model. Assuming \( p, \) the order of linear AR and the delay parameter \( d \) are known, the auxiliary regression to be estimated is

\[
\begin{align*}
  z_t &= \alpha_0 + \sum_{i=1}^{p} \alpha_i z_{t-i} + \sum_{i=1}^{p} \alpha_i^* z_{t-d-i} + \sum_{i=1}^{p} \hat{\theta}_i z_{t-d-i}^2 + \omega_t, \\
  \end{align*}
\]

(4)

where \( \alpha_0, \alpha_i, \alpha_i^* \) and \( \hat{\theta}_i \); \( i = 1, \ldots, p \) are parameters to be estimated and under the null hypothesis, \( \omega_t \) is simply a white noise with zero mean and constant variance.

The linearity test as given by Equation 4 assumes that if the null hypothesis of linearity is rejected in favour of the alternative, then the ESTAR model is the correct specification. Otherwise, the joint effect of \( \alpha_i^* \) and \( \hat{\theta}_i \) in Equation 4 would effectively be zero. To sum, the test may be conducted by the following steps:

(i) Regress \( \{ z_t; t = 1, \ldots, T \} \) on \( \{ 1, z_{t-1}, \ldots, z_{t-p} \} \), form the residuals \( \{ \hat{e}_t, \ldots, \hat{e}_T \} \) and obtain the residual sum of squares, \( \text{SSR}_0 = \sum_{t=1}^{T} \hat{e}_t^2 \);

(ii) Regress \( \{ \hat{e}_t; t = 1, \ldots, T \} \) on \( \{ 1, z_{t-1}, \ldots, z_{t-p}, z_{t-d}, \ldots, z_{t-p}z_{t-d}, z_{t-d}^2 \} \), form the residuals \( \{ \hat{\omega}_t, \ldots, \hat{\omega}_T \} \) and obtain \( \text{SSR}_f = \sum_{t=1}^{T} \hat{\omega}_t^2 \);

(iii) Compute the two tests statistics:

\[
\begin{align*}
  \text{LST} &= \frac{T(\text{SSR}_0 - \text{SSR}_f)}{\text{SSR}_0} a\tilde{y}^2 \chi^2_{p+1}; \\
  \text{LM} &= \frac{\text{SSR}_0 - \text{SSR}_f}{\sigma_e^2} a\tilde{y}^2 \chi^2_{2p} \quad \text{where } \sigma_e^2 \text{ is the variance of } \hat{e}_t
\end{align*}
\]

(5) (6)

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\(^8\) We design computer algorithms for this estimation using the Regression Analysis of Time Series (RATS) software, with the initialization of \( \beta_0 = \gamma = \beta_i = \beta_i^* = 0.01; i = 1, \ldots, p \). The stopping criterion is that the neighbouring residual sum of squares is less than or equal to 0.00001. Taylor and Peel (2000) exert that estimation with various starting values for the parameters in fact yielded identical results if the global minimum of the criterion function exists. As for \( \sigma_e^2 \), it is easily estimated from the residuals of \( z_t \).
The LST and LM statistics under the null hypothesis are distributed asymptotically as a chi-squared with \( p+1 \) and \( 2p \) degree of freedom respectively. The rejection of the above test statistics implies nonlinearity and hence is in favour of ESTAR. The optimum lag length \( p \) is usually unknown even if the true model is linear and has to be determined from the data. In this study, the optimal lag length \( p \) of linear AR (\( p \)) model is selected by using the Akaike’s biased corrected information criterion (AICC). A recent study by Liew and Shitan (2002) that examined the behaviour of AICC through a simulation exercise found that among a class of models with no serial correlation, minimizing AICC has little tendency to underestimate the true order \( p \). Hence, following their suggestion we used AICC to avoid the problem of too parsimonious a model being selected.\(^9\)

Having selected \( p \), \( d \) needs to be determined. In order to specify \( d \), the test is carried out for the range of values \( 1 \leq d \leq 5 \) considered appropriate. If the linearity is rejected for more than one value of \( d \), then \( d \) is determined as \( \hat{d} = \arg \min p(d) \) for \( 1 \leq d \leq 5 \) where \( p(d) \) is the \( p \) value of the selected test. The argument behind this rule of using the marginal significant rule is that the test has maximum power if \( d \) is chosen correctly, whereas an incorrect choice of \( d \) weakens the power of the test. The Ljung-Box portmanteau [Q(20)] test is also employed to confirm the absence of serial correlation up to 20 lags.

**Forecasting models**

We incorporate the unrestricted and restricted ESTAR models into the long-run equilibrium PPP model to generate future values of MYR/JPY rate. The resulting unrestricted and restricted forecasting models are, in that order, given as:

\[
\hat{X}_{t+n} = \hat{X}_{t+n} + \hat{\beta}_0 + \sum_{i=1}^{p} \hat{\beta}_i \hat{z}_{t+n-i} + \left[ \sum_{i=1}^{p} \hat{\beta}_i \hat{z}_{t+n-i} \right] \left[ 1 - \exp \left(-\left(\gamma^2 / \sigma_z^2\right)z_{t-d}^2 \right) \right] \tag{7}
\]

and

\[
\hat{X}_{t+n} = \hat{X}_{t+n} + \left[ \sum_{i=1}^{p} \psi_i \hat{z}_{t+n-i} \right] \left[ \exp \left(-\left(\gamma^2 / \sigma_z^2\right)z_{t-d}^2 \right) \right] \tag{8}
\]

where \( \hat{X}_{t+n} \) stands for combined forecasts of the linear and nonlinear models and \( n \) is the forecast horizon. \( \hat{X}_{t+n} \) is the forecasts of fundamental models, in particular the PPP model and \( \hat{z}_{t+n} \) represents the forecasted residuals of the ESTAR model. \( \gamma^2, \sigma_z, \hat{\beta}_0, \hat{\beta}_i \) and \( \hat{\psi}_i; i = 1, \ldots, p \) are estimated parameters.

**Criteria for evaluation of forecasting performance**

The overall in-sample and out-of-sample performance of the estimated forecasting models over the forecast horizon of \( n = 14 \) over the period 1997:3 to 2000:4 are evaluated by using the linear AR (\( p \)) and random walk (RW) models as the benchmark. The criteria involved are the ratios of forecast error, which is measured by root mean squares forecast error (RMSE), mean absolute deviation

\(^9\) Several order selection criteria are employed for the same purpose in the previous related studies. For instance, Luukkonen et al. (1988) use SIC, while Teräsvirta and Anderson (1993) use the AIC criterion. However SIC and AIC are well known for selecting too parsimonious a model that is not serial correlation free. Taylor and Peel (2000) and Sarno (2000) select \( p \) based on the partial autocorrelation functions (PACF) of \( z_t \), Teräsvirta (1994) and Baum et al. (2001) use the Ljung-Box portmanteau (Q) test.
(MAD) and mean absolute percentage error (MAPE), of the two competing models, with the forecast error of the benchmark model as denominator.

Unlike prior studies, we also test for the statistical significance of these ratios. In fact, Mizrach (1995) has documented that heuristic approaches of ratio analysis often results in misleading inference. Hence, to avoid this shortcoming, we provide a statistical test as a complement to the ratio analysis. Specifically, we compute the Fisher’s Sign (FS) test statistics for the overall in-sample and out-of-sample forecasts. The Fisher’s sign test is the total number of negative lost differential \(d_j\) observations in a sample size \(n\). The null hypothesis is that the ESTAR model and the random walk (or AR) model provide equally accurate forecasts. The alternative hypothesis is that our forecasting model has better accuracy than the benchmark. Under the null hypothesis of equal accuracy of two competing forecasts, the FS has a binomial distribution with parameter \(n\) and 0.5. The significance of test is assessed using a table of the cumulative binomial distribution provided in Keller and Warrack (1997, pp. 522-528).

**EMPIRICAL RESULTS AND INTERPRETATIONS**

**Linearity tests results**

The results of the linearity tests are summarized in Table 3. It is clear from this table that linearity is rejected for the PPP-based residuals in favour of the ESTAR model. Notice that the optimal values of \(p\) and \(d\) are determined as 2 and 1 respectively. This implies the adjustment of deviations towards the long-run PPP equilibrium of MYR/JPY rate follows the ESTAR (2) process with a delay parameter, \(d = 1\).

<table>
<thead>
<tr>
<th>Test statistics</th>
<th>Delay parameter, (d)</th>
<th>Marginal significance value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LST test</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>LM test</td>
<td>1</td>
<td>0.001</td>
</tr>
<tr>
<td>Ljung-Box Q statistic</td>
<td>1</td>
<td>0.722</td>
</tr>
</tbody>
</table>

Notes: The optimal lag length \(p\) of linear AR \((p)\) model is determined as 2 by the AICC. Optimal delay parameter \(d\) is determined by \(d = \arg \min \ p(d), \ i \in \{\text{LST, LM}\}\), where \(p(\cdot)\) denotes the p-value of the implied test statistic for the null hypothesis of \(H_0: \text{Linear model is correct. Rejection of } H_0 \text{ implies nonlinearity in favour of the ESTAR model.}

**Estimated ESTAR models**

Results of unrestricted and restricted ESTAR models are summarized in Table 4. From Equation 9 in Table 4, it is clear that the residual variance ratio of the unrestricted ESTAR (2) model to the linear AR (1) model is 0.769, indicating that the former has a much smaller variance. Thus, the nonlinear model has the ability to produce smaller forecast errors than the linear model. The marginal significance value of the Ljung-Box Q(20) statistic, suggests that this model is free from serial correlation up to 20 lag length. A conclusion that can be drawn from these two diagnostic tests is that the nonlinear model is an appropriate representation of the PPP-based residuals. The unstandardized and standardized transition parameters are 1.364 (significant at 1% level) and 10.880 respectively. This finding suggests that the speed of transition between different regimes is fairly rapid.

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10 If the ratio is greater than one, it implies the benchmark model is better. If the ratio is less than one, it means the forecasting model has defeated the benchmark model and the researchers’ effort is at least paid-off. It is worth to note that the closer the ratio is to zero, the more excellent is the forecasting model.
TABLE 4.
Estimated Estar Models

<table>
<thead>
<tr>
<th>Equation</th>
<th>Unrestricted ESTAR (2) model</th>
<th>Restricted ESTAR (2) model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{z_t} = -0.031 + 1.799 z_{t-1} - 0.340 z_{t-2} + \left[ -1.170 z_{t-1} + 0.349 z_{t-2} \right] \times \left[ 1 - \exp \left( -\left(1.364^2 / 0.171\right) \times z_{t-1}^2 \right) \right] )</td>
<td>( \hat{z_t} = -0.025 + \left[ 1.442 z_{t-1} - 0.253 z_{t-2} \right] \times \left[ \exp \left( -\left(0.357^2 / 0.171\right) \times z_{t-1}^2 \right) \right] )</td>
<td></td>
</tr>
<tr>
<td>(0.19) (0.33)** (0.23) (0.40)** (0.50) (0.25)**</td>
<td>(0.02) (0.19)** (0.16) (0.05)**</td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma}_{ESTAR}^2 = 0.002 )</td>
<td>( \hat{\sigma}_{ESTAR}^2 = 0.002 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{\hat{\sigma}<em>{ESTAR}^2}{\hat{\sigma}</em>{AR}^2} = 0.769 )</td>
<td>( \frac{\hat{\sigma}<em>{ESTAR}^2}{\hat{\sigma}</em>{AR}^2} = 0.819 )</td>
<td></td>
</tr>
<tr>
<td>Q(20) = 17.522 [0.619]</td>
<td>Q(20) = 20.492 [0.428]</td>
<td></td>
</tr>
<tr>
<td>( R^2 = 0.882 )</td>
<td>LR = 8.952[0.030]</td>
<td>( R^2 = 0.878 )</td>
</tr>
</tbody>
</table>

Notes: \( \hat{z_t} \) is the estimated \( z_t \), the PPP-based residuals of MYR/JPY rate. \( \hat{\sigma}_{ESTAR}^2 \) and \( \hat{\sigma}_{AR}^2 \) are the estimated variance of the ESTAR model and the AR model respectively. \( R^2 \) stands for adjusted \( R^2 \).

The likelihood ratio (LR) test is the test statistic for the restrictions implicit in the estimated equation, against the corresponding unrestricted ESTAR model (see, Taylor and Peel, 2000). LR is chi-squared distributed with degree of freedom equals number of restrictions. Values in parentheses and brackets refer to standard errors of estimated coefficients and marginal significance values respectively. Standard error with ** and * denote the estimated coefficient is significant at 1% and 5% level respectively.

Figure 1 plots the estimated exponential transition function, \( F(z_{t-1}) \) against the corresponding deviations of MYR/JPY from the long-run PPP equilibrium, \( z_{t-1} \). It is clear from the plot that the distribution of this exponential \( F(z_{t-1}) \) follows the typical inverted bell-shape. For the magnitude of \( z_{t-1} \) below 50% of the deviations from the PPP equilibrium, the slope of \( F(z_{t-1}) \) is fairly steep (45º). Thus, the dynamic adjustment is rather highly rapid and it explodes to the second order unit root process in the outer regime as the absolute size of deviations from equilibrium reaches beyond 0.50 in value.

The restricted version of the ESTAR (2) model in Equation 10 is also estimated. As discussed earlier, the restrictions imposed on this model are \( \beta_0 + \beta_1 + \beta_2 = 1, \beta_1^* = -\beta_1, \beta_2^* = -\beta_2 \). Testing for these restrictions are straightforward using the log-likelihood ratio (LR) test. The test statistic is distributed as chi-squared with 3 degrees of freedom. The computed value of LR statistic suggests that the above restrictions cannot be rejected at the 5% significance level. Besides that, the restricted ESTAR (2) model passes a battery of diagnostic test. The model representation is given in Equation 10 in Table 4. The variance ratio of 0.819 in Equation 10, suggests that the restricted ESTAR (2) model is also smaller than that of the linear version. Note that the estimated transition parameter \( \gamma^2 \) of 0.357 indicates significance at the 1% level.
Unrestricted ESTAR model

\[
F[z(t-1)] = 1 - \exp\left(-\frac{\gamma^2}{\sigma^2} z(t-1)^2\right)
\]

A plot of the estimated exponential transition function, \( F(z_{t-2}) \), is also depicted in Figure 1. The transition function of the restricted ESTAR model attains values as high as 0.50, a value half of the unrestricted transition function’s maximum value. Furthermore, for the positive and negative deviations from the fundamental equilibrium of magnitude 0.50, the values of the transition function clustered around zero, indicating that the adjustment process of the exchange rate deviation to the equilibrium almost reverted to the linear AR (2) model in the outer regime of Equation 10.

Forecasting performance of estimated models

Our ultimate unrestricted and restricted forecasting models are, in that order

\[
\hat{X}_{t+1} = -0.031 + \hat{X}_{t+1} + 1.799 z_{t+1} - 0.340 z_{t+2} + (-1.170 z_{t+1} + 0.349 z_{t+2}) \times [1 - \exp((-0.817/0.171) \times z_{t-1}^2)]
\]

and

\[
\hat{X}_{t+1} = -0.025 + \hat{X}_{t+1} + [1.442 z_{t+1} - 0.253 z_{t+2}] \times [\exp((-0.357/0.171) \times z_{t-1}^2)]
\]

The resulting models are used to generate the in-sample as well as the out-of-sample forecasts and the forecasting performances of these two models are evaluated. The results of the forecasting performances are displayed in Table 5. For the in-sample forecasts, the results of the RMSE, MAD and MAPE criteria consistently show that both the unrestricted and restricted ESTAR models are capable of generating better forecasts over the linear AR models (significant at 5% level by the FS test). However, as revealed by the FS statistics, the in-sample performances of both ESTAR models are insignificantly different from the random walk model. This means that although the nonlinear model’s predictions dominate the random walk in RMSE, MAD and MAPE, the improvement in prediction accuracy is insignificant even at the 10 percent significant level.
### TABLE 5

**Overall performances of forecasting models**

<table>
<thead>
<tr>
<th>Forecast horizon</th>
<th>In-sample (n = 65)</th>
<th>Out-of-sample (n = 14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Models</td>
<td>Unrestricted</td>
<td>Restricted</td>
</tr>
<tr>
<td>Linear AR (2) model as benchmark</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSE ratio</td>
<td>0.824</td>
<td>0.854</td>
</tr>
<tr>
<td>MAD ratio</td>
<td>0.770</td>
<td>0.820</td>
</tr>
<tr>
<td>MAPE ratio</td>
<td>0.779</td>
<td>0.823</td>
</tr>
<tr>
<td>Fisher’s sign test</td>
<td>40 [0.023]</td>
<td>42 [0.027]</td>
</tr>
<tr>
<td>Random walk model as benchmark</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSE ratio</td>
<td>1.205</td>
<td>1.249</td>
</tr>
<tr>
<td>MAD ratio</td>
<td>1.085</td>
<td>1.156</td>
</tr>
<tr>
<td>MAPE ratio</td>
<td>1.135</td>
<td>1.199</td>
</tr>
<tr>
<td>Fisher’s sign test</td>
<td>28 [0.840]</td>
<td>34 [0.500]</td>
</tr>
</tbody>
</table>

*Notes*: The null hypothesis of Fisher’s sign test is 2 models have equal forecast accuracy against the alternative hypothesis that forecasting model predicts more accurately than the benchmark model. Total numbers of negative lost differential is reported with marginal significance value given in brackets. Lost differential = \( \frac{SE_{FM,j} - SE_{BM,j}}{SE_{FM,j}} \), where \( n \) is the forecast horizon. \( SE_{FM} \) and \( SE_{BM} \) stand for forecasting model and benchmark model respectively.

In the out-of-sample forecasting, the RMSE ratios are 0.880 and 0.856, respectively, when the AR (2) and RW models are used as the benchmark for comparison. This finding implies that the unrestricted model outperforms both benchmark models. A similar conclusion can be made using the MAD and MAPE criteria. Nonetheless, the FS test results suggest that this improvement is not statistically significant at 10% level. As for the restricted model, the predictions are significantly more precise than the linear AR (2) model (significant at 5% level) and the random walk model (significant at 10% level). To sum up, our forecasting models are able to predict better than both the AR and RW models up to 14 quarters of forecast horizon. All in all, the restricted ESTAR model yields more accurate forecasts than the unrestricted ESTAR model.

### CONCLUDING REMARKS

The purpose of this paper is to contribute to the debate on the relevance of nonlinear models in currency markets. To that end, we employed the ESTAR model to show that the reason for poor predictive performance of exchange rate models is due to nonlinearity the adjustment of exchange rate to its long-run equilibrium path. For the Middle Eastern countries, Sarno (2000) demonstrates that the real exchange rate is well characterized by a process, which adjusts nonlinearly toward its long-run equilibrium but the ESTAR model fits in only 3 (Lebanon, Iran and Turkey) out of the 11 countries selected for the study.

The main findings of this study are summarized as follows. First, data for the MYR/JPY rate supports the existence of nonlinearities. The data supports long-run PPP and there is strong statistical evidence to suggest that exchange rate adjusts toward its long-run equilibrium in a nonlinear fashion. Second, the ESTAR models produce smaller out-of-sample forecast errors than those of random walks or AR model. We find that linear models are not always the optimal for forecasting exchange rate. Not surprisingly, the hypothesis of equal forecasting accuracy between ESTAR models and the random walk model is formally rejected based on Fisher’s sign test. In other words, the superiority of the ESTAR forecasts is statistically significant from the linear benchmark models. Taken together, the evidence presented in this article confirms the results of recent studies that emphasize the importance of allowing nonlinearity in the adjustment of exchange rates towards its long-run equilibrium path, hence offering a potential reason why PPP or its variants may failed to out-perform the forecasting performance of the random walk model.
REFERENCES


